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43. QUANTUM CHAOS IN TWO-LEVEL SYSTEMS INTERACTING WITH RESONANT PERIODIC OSCILLATING FIELDS

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1. Introduction

For atomic radiation systems resonantly interacting each other, it is seen that chaotic atomic motion appears when coupling between atoms and radiation is strong enough or a certain detuning frequency exists.[1] That was investigated numerically in the frame of semiclassical treatment, which is based on Maxwell-Bloch type equations, in which no pump is introduced. Fully quantum treatments are now tried for this closed systems which have definite hamiltonian of atomic and radiation field, without external force like a pumping et.al..

2. Quantum resonant systems

We construct quantum mechanical systems corresponding to the former systems treated classically. Spin state is represented by the atomic coherent states, in which operators are described by polar coordinate of the Bloch space, which is similar to space of top motion.[2] Field state is now represented by coherent states, which corresponds to classical field amplitude E et.al., where the annihilation and creation operators are described by amplitude variables.

Interaction hamiltonian is expressed by the operators and coupling constant μ , as follow;

$$\mathcal{H}_I = i\hbar\mu (a^\dagger S - S^\dagger a - S a^\dagger + a S^\dagger) \quad (1)$$

where we did not take the rotating approximation, by which the first two terms would have been left, that would make a drastic change to the chaotic behavior of the systems. Distribution function is used for describing the behavior of systems. Variable of that function are expectation value of quantum variable, that is, angular variables of Bloch space and field amplitude. That is obtained by expanding density operator, which is expanded by complete set of atomic coherent and field coherent states, whose coefficients are defined to be that distribution function.

$$\rho = \int d\Omega \int d^2E P_c(\theta, \phi, E) |\Omega\rangle\langle\Omega| |E\rangle\langle E| \quad (2)$$

After tedious operator algebra we can obtain the partially differential equation for systems described quantum mechanically.

$$\begin{aligned} \frac{\partial P_c}{\partial t} = & -\omega_c \left(\frac{\partial}{\partial E_x} E_x - \frac{\partial}{\partial E_y} E_y \right) P_c \\ & + 2\mu E_x \left\{ \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta P_c) - \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \varphi} (\sin \theta P_c) \right\} - 2\mu E_y \left\{ \frac{\sin \theta}{\sin \theta} \frac{\partial}{\partial \theta} (\sin \theta P_c) - \frac{\cos \theta}{\sin \theta} \frac{\partial}{\partial \varphi} (\cos \theta P_c) \right\} \\ & - \mu^2 E_x \left\{ (1+i) \sin \theta e^{-i\varphi} + \frac{1-\cos \theta}{2} e^{-i\varphi} \frac{\partial}{\partial \theta} + i \frac{1-\cos \theta}{2} e^{-i\varphi} \frac{\partial}{\partial \varphi} \right\} P_c + \mu^2 E_y \left\{ (1-i) \sin \theta e^{-i\varphi} + \frac{1-\cos \theta}{2} e^{-i\varphi} \frac{\partial}{\partial \theta} + i \frac{1-\cos \theta}{2} e^{-i\varphi} \frac{\partial}{\partial \varphi} \right\} P_c \quad (3) \end{aligned}$$

In order to solve this equation numerically we expand the unknown function by complete set of eigenfunctions, which are selected to be associate legendre functions for bloch space variable θ and φ , and hermite functions for the two dimensional space of electrical field variable, and as follows;

$$P_c = \sum_{l=0}^{\infty} \sum_{m=-l}^l \sum_{n_x} \sum_{n_y} i^m a_{lmn_xn_y} Y_{lm}(\theta, \varphi) H_{n_x}(E_x) H_{n_y}(E_y) \quad (4)$$

The expansion coefficients are now the unknown variables calculated numerically. It may be an advantage of those choice of eigen functions that the 1st order coefficients of legendre series with 0th order of hermit functions and the 1st order coefficients of hermite series with 0th of legendre ones are equal to the classical expectation values of spin variables and field ones respectively, and further the associate legendre series terminate in a finite terms because the associate legendre functions is just the eigen functions of spin systems which is represented by bloch space.

As the above eq.(3) is linear partially differential equation, the equations of unknown coefficients have the form of linear ordinary differential equations with infinite unknowns, and constant coefficient, that might suggest that we could solve it by numerical linear transformation. But very large area of memory is needed for constructing the matrix, so we now solve those by Runge-Kutta formula of initial value problems. The above quantum system was, however, formally converted to a linear systems with infinite dimensions, where classical behavior was include as the lowest order partial space which cannot, however, be treated reconnected with the whole space.

4. Motions of classical variables

Numerical values of 1st order coefficients of either legendre series or hermite ones represent the motion of corresponding classical variables. Results were obtained for the case of various

coupling constant which make it possible to compare with the case of classical theory. Fig.1 shows the variation of expected spin variable and electrical field, and their x-y trajectory, for the same conditions as classical ones. Time variation may be seen to be high cut-offed filtered, that was also shown by their fourier spectrum which was simply obtained by fft programs for both semiclassical and quantum theoretical cases, shown in fig.1. It has been observed in many models of chaotical systems that chaos is suppressed when classical systems were converted to corresponding quantum systems, that reveals in this case, where chaotic motion may not become extinct but may be hidden somewhere in the systems, where classical variable shown here, were included in only a partial space. In the case of stronger coupling constant, the same motion and its spectrum were represented, as the classical chaotic phenomena, as shown in fig.1.

Now we see the quantum parameter which was included in higher terms than 1st, but distribution function also shows the whole structure. Therefor, using the all coefficients, we calculate this function and show it by graphical technique, which were shown in fig.2, where the appearance of function is seen to correspond to the classical case more than the 1st order variables, that is, classical chaotic structure in the quantum system, appears in the shape of distribution functions, which is equivalent to wave functions through the conversion of representation, not in the 1st order parameter. Complexity of two dimensional function should be measured quantitatively.

In fig.2 the contour lines are also represented for the respective distribution functions, and in fig.2 ones of the other case are also shown. The analogy with spin systems of ferro-magnetism, suggest that the distribution will have self-similar or fractal patterns when the chaotic motion of system is occurring. Fractal dimensions obtained from the respective contour lines are nearly equal to an integer numbers and changed for all the cases of coupling constant, that shows that fractal dimension does not represent the complexity of distribution function, for which we must find the proper parameter to estimate the complexity of curved surface.

References

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 [2] G.S.Agarwal et.al.: Phys. Rev. A 20(1979)2040.

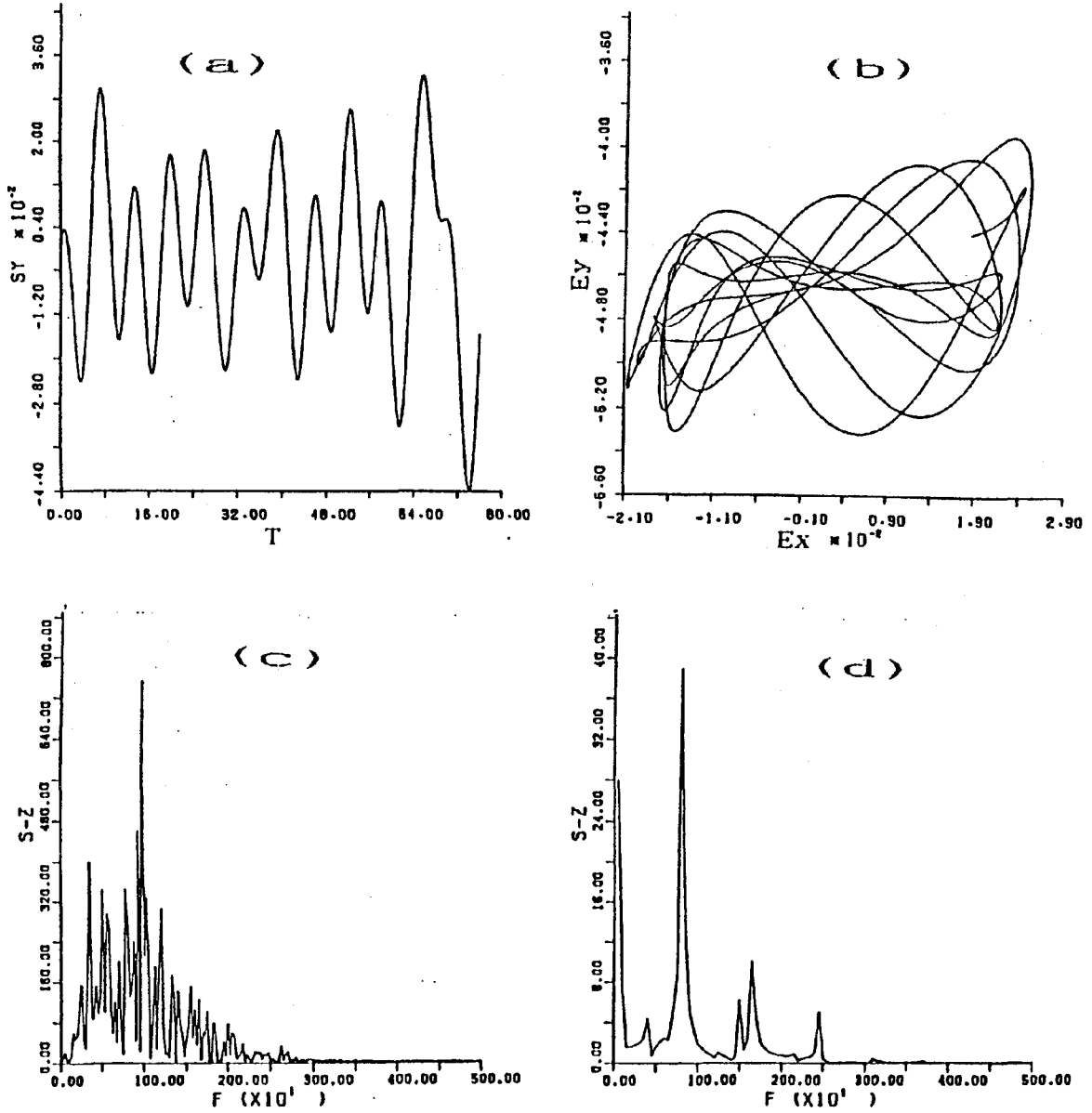


Fig.1 Variation, x-y trajectory and its fourier spectrum of spin variables and electric field. $\omega_c=17711/28657$, $\omega_a=1$. (a), (b) $\mu=0.2$, (c) spectrum of semiclassical variable $\mu=0.5$, (d) spectrum of quantum results $\mu=0.5$.

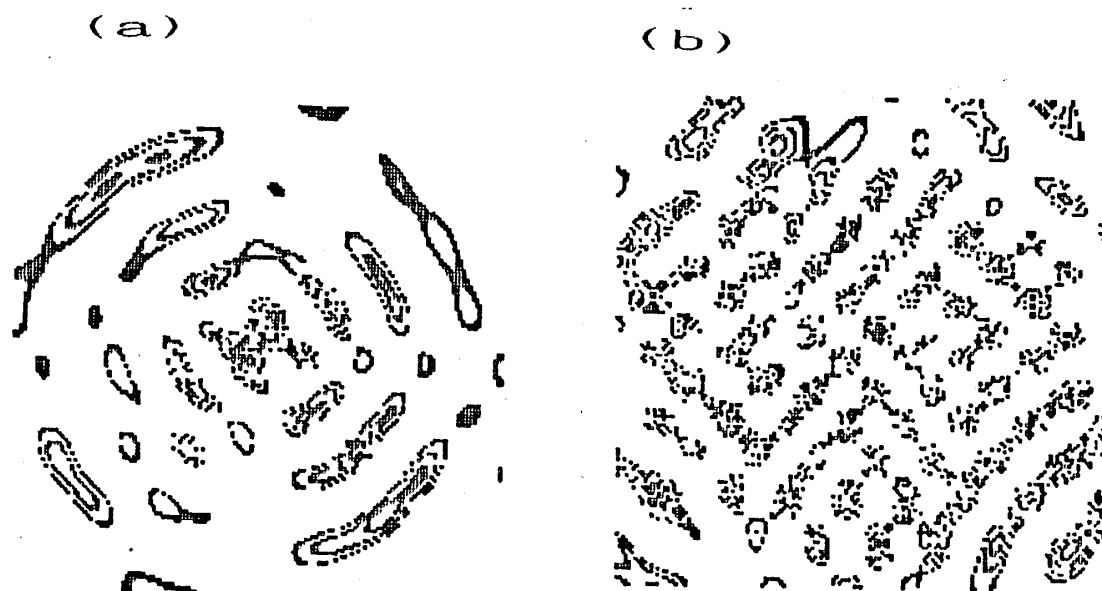


Fig.2 C-number distribution function and their contour lines of expected value of quantum electric field variable, at $t=40$.

(a) $\mu=0.2$. (b) $\mu=1.0$

4.5. 量子カオスの情報論的アプローチ

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非可積分量子系の波動関数は、その節線 (Nodal line) の非交叉に特徴づけられる複雑なパターンを示し、系がカオス的になる程、その乱れは大きくなる。ここでは、以下で定義される正準相関とよばれる情報理論的な量を用いて、非可積分系における固有関数の定量的な特徴づけを試みる。

まず、2つの部分系 S_1 , S_2 からなる結合系 $S = S_1 + S_2$ を考え、結合系 S の任意の状態 ϕ^S を、それぞれ S_1 , S_2 の完全直交基底 $\{\phi_i\}$, $\{\theta_j\}$ からつくられる直積基底を用いて、

$$\phi^S = \sum_{ij} a_{ij} \phi_i \theta_j \quad (1)$$

として表わす。部分系 S_1 と S_2 の間に全く相互作用がなく独立な場合、

$$\phi^S = \left(\sum_i c_i \phi_i \right) \left(\sum_j d_j \theta_j \right) = \left(\sum_{ij} c_i d_j \phi_i \theta_j \right) \quad (2)$$